ALLAMA IQBAL OPEN UNIVERSITY, ISLAMABAD
(Department of Mathematics and Statistics)

WARNING

1. **PLAGIARISM OR HIRING OF GHOST WRITER(S) FOR SOLVING THE ASSIGNMENT(S) WILL DEBAR THE STUDENT FROM AWARD OF DEGREE/CERTIFICATE, IF FOUND AT ANY STAGE.**
2. **SUBMITTING ASSIGNMENTS BORROWED OR STOLEN FROM OTHER(S) AS ONE'S OWN WILL BE PENALIZED AS DEFINED IN "AIOU PLAGIARISM POLICY".**

Course: Discrete Mathematics (3406) **Semester: Autumn, 2012**
Level: BS (CS) **Total Marks: 100**
Pass Marks: 40

ASSIGNMENT No. 1
(Units 1–4)

Note: Attempt all questions and each question carries equal marks.

Q.1 (a) Use truth table to establish which of the following statements are tautologies and which are contradictions? (10+10)

i. $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$

ii. $(\sim p \vee q) \vee (p \wedge \sim q))$

(b) For the table given below;

P	Q	R	S
1	1	1	0
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	0
0	1	0	1
0	0	1	0
0	0	0	0

i. A Boolean expression having the given table as its truth table.
ii. A circuit having the given table as its input / output table.

Q.2 (a) Prove modus tollens. In other words, prove that the following argument form is invalid: (5+5+5+5)

$$p \rightarrow q$$
$$\sim p$$
$$\therefore \sim q$$

(b) Write negation of the statement:
$\forall$*animals x; if x is a cat then x has whiskers and x has claws.*

(c) Use modus and tollens to fill in valid conclusion for the following argument;
All healthy people eat an apple a day.
Harry does not eat an apple a day.
$\therefore$ _____ .

(d) Indicate whether the following arguments are valid or invalid? Support your answer by drawing diagrams.
All people are mice.
All mice are mortal.
$\therefore$ All people are mortal.

Q.3 (a) Use symbols to write the logical form of the following arguments then use a truth table to test the arguments for validity; (5+5+10)
If Tom is not on team A, then Hua is on team B.
If Hua is not on team B, then Tom is on team A.
$\therefore$ Tom is not on team A or Hua is not on team B.

(b) What can you conclude about the validity OR invalidity of the following argument form?

$\forall x$, *if P (x) then Q(x);*
$\sim P(a)$ For a particular α
$\therefore \sim Q(a)$

Q.4 (a) Prove that for all positive integers a and b, $a \mid b$ if, and only if, gcd $(a, b) = a$. (5+5+10)

(b) For all integers a, b and c, if $a \mid b$ and $a \mid c$ then $a \mid (b + c)$.

(c) Use the well-ordering principal to prove that if a and b are any integers not both zero then there exist integers u and v such that gcd$(u, v) = ua + vb$.

Q.5 (a) Write an algorithm to determine whether a given element x belongs to a given set, which is represented as an array $a[1], a[2], a[3], \ldots, a[n]$. (10+10)

(b) Write a negation for the following statements;
i. $\forall$ Sets S, $\exists$ a set T such that $S \cap T = \varnothing$. Which is the true statement or its negation? Explain.

ii. $\exists$ a Set S *such* that $\forall$ sets T, $S \cup T = \varnothing$. Which is the true statement or its negation? Explain.

ASSIGNMENT No. 2

(Units 5–9)

Total Marks: 100 **Pass Marks: 40**

Note: Attempt all questions and each question carries equal marks.

Q.1 (a) Suppose that there are three roads from city A to city B and five roads from city B to city C; (10+10)

i. How many ways is it possible to travel from city A to city C via city B?

ii. How many different round trip routes are there from city A to B to C to B and back to A in which no road is traversed twice?

(b) A computer programming team has 14 members.

i. Suppose two team members refuse to work together on project. How many groups of 7 can be chosen to work on a project?

ii. How many ways can a group of 7 are chosen to work on a project?

iii. Suppose 8 team members are women and 6 are men; how many groups of 7 can be chosen that contain 4 women and 3 men?

Q.2 (a) Draw arrow diagrams for the Boolean functions defined by the following input / output table; (10+10)

Input		**Output**
P	**Q**	**R**
1	1	0
1	0	1
0	1	0
0	0	1

(b) Given any set of seven integers;

i. Must there be two that have the same remainder when divided by 6? Why?

ii. Must there be two that have the same remainder when divided by 8? Why?

Q.3 (a) A single pair of rabbits (Male & Female) is born at the beginning of a year. Assume the following conditions; (10+10)

i) Rabbit pairs are not fertile during their first two months of life, but there after gave birth to three new male/female pairs at the end of every month;

ii) No deaths occur during the year.

1. Let S_n the number of pairs of rabbits live at the end of month n, for each integer $n \geq 1$, and let $S_0 = 1$. Find a recurrence relation for $S_0, S_1, S_2,...$
2. Compute S_0, S_1, S_2 and S_4
3. How many rabbits will be there at the end of the year?

(b) A runner targets herself to improve her time on a certain course by 3 seconds a day. If on day 0 she runs the course in 3 minutes, how fast must she run it on the 14^{th} day to stay on target?

Q.4 (a) Show that for any real number x, if $x > 1$ then $|2x^2 + 15x + 4| \leq 21 |x^2|$ and use O- notation to express the result? (10+10)

(b) Refer to the following algorithm segment. For each positive integer n, let b_n be the number of integrations of the while loop;

While (n > 0)
***n* := *n div* 3**
End while

Trace the action of the algorithm segment on n when the initial value of n is 424.

Q.5 (a) Draw all non-isomorphic graphs with four vertices and no more than two edges. (5+5+10)

(b) Prove that if a walk in a graph contains a repeated edge, then the walk contains a repeated vertex.

(c) For the following graph determine whether there is an Euler path from U *to w.* If there is, find such a path: